MICROWAVE FILTER DESIGN IN THE TIME DOMAIN

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ABSTRACT

Commensurate transmission-line networks are designed in the time domain using state-space techniques with no restrictions on the network topology. Computer-aided procedures are used to optimise the time domain responses. Several examples are given of microwave networks designed using this technique.

Introduction

State-space techniques are used in this work to analyze commensurate microwave networks of any topology in the time domain. The reflection coefficient is calculated and matched to the desired ideal function. A computer-aided procedure calculates the values of the circuit elements that minimise the difference between the ideal and circuit responses. Conditions on both the amplitude and phase responses of the network can be transferred to the time domain and the circuit can be designed to meet these requirements together with any other constraints imposed on the time domain response. Since the time response is a function of a real variable t instead of the complex frequency variable s, the computer-aided procedure is very efficient compared with procedures optimising the responses in the frequency domain.

The Network Response

Commensurate networks can be analyzed in the time domain using state-space techniques *[1-2]*. The state and output equations are given by

$$x(t + T) = A\,x(t) + B\,u(t) \tag{1a}$$

$$y(t) = C\,x(t) + D\,u(t) \tag{1b}$$

where x(t), u(t) and y(t) are the state, input and output vectors respectively and T is the commensurate delay on each line. The matrices A, B, C and D are derived *[1-2]* in terms of the network topology and the element values. With this method there are no restrictions on the network topology and coupling between the lines can be included.

The response h(t) for a unit impulse input δ(t) is derived from (1) and is given by

$$h(t) = \sum_{j=1}^{\infty} C\,A^{j-1}\,B\,\delta(t - jT) + D\,\delta(t) \tag{2}$$

The output could be chosen as the voltage drop $v_{R1}(t)$ across the generator resistance R_1 and the matrices C and D are derived accordingly. In this case $v_{R1}(t)$ will be given by (2) and the reflection coefficient $S_{11}(t)$ in the time domain will be given by

$$S_{11}(t) = \delta(t) - 2v_{r1}(t) \tag{3}$$

If the output is chosen as the voltage v_{R2} across the load resistance R_2 then the matrices C and D are derived accordingly and v_{R2} will be given by (2). The transfer scattering parameter $S_{21}(t)$ in the time domain will be given by

$$S_{21}(t) = \sqrt{\frac{R_1}{R_2}}\;v_{R2}(t) \tag{4}$$

The forms of equations (1) and (2) do not change by a different choice of output; only the matrices C and D will depend on that choice.

The above analysis gives a very general method of calculating the time domain response of commensurate networks of any topology.

Ideal Response

The ideal response can be either of the scattering parameters $S_{11}(t)$ or $S_{21}(t)$. In this work we will give examples of networks designed to approximate an ideal reflection coefficient $S_{11}(t)$. The form of $S_{11}(t)$ will be derived from the ideal return loss function L_R shown in Figure 1.

The ideal return loss L_R is related to the ideal reflection coefficient in the frequency domain $S_{11}(j\omega)$ by

$$L_R = \ln \frac{1}{|S_{11}(j\omega)|} \tag{5}$$

Since L_R is a periodic function of frequency, it can be expanded in a Fourier series *[3]*

$$L_R = a_o/2 + \sum_{r=1}^{\infty} a_r \cos(2r\omega T) \tag{6}$$

For the ideal low-pass distributed filter response shown in Figure 1(a), a_o and a_r are given by

$$a_o = 2A\omega_c T/\pi \tag{7a}$$

$$a_r = 2A\sin(2r\omega_c T)/\pi r \tag{7b}$$

where A and ω_c are the specified return loss in the pass band and the cut-off frequency respectively. For the ideal band-pass response shown in Figure 1b, a_o and a_r are given by

$$a_o = A(1 - 2\omega_c T/\pi) \tag{8a}$$

$$a_r = -\,2A\sin(2r\omega_c T)/\pi r \tag{8b}$$

The ideal minimum phase reflection coefficient can be written as

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$$S_{11}(z) = \exp(-a_0/2)\{1 + \sum_{r=1}^{\infty} b_r z^{-2r}\} \qquad (9)$$

where $z = \exp(sT)$
and

$$b_r = \frac{1}{r!} \begin{vmatrix} a_1 & -1 & 0 & 0 \ldots 0 \\ 2a_2 & a_1 & -2 & 0 \ldots 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ ra_r & (r-1)a_{r-1} & \ldots\ldots\ldots\ldots & a_1 \end{vmatrix} \qquad (10)$$

In the time domain $S_{11}(t)$ is obtained through the inverse Laplace transform of (9)

$$S_{11}(t) = \exp(-a_0/2)\delta(t) + \exp(-a_0/2) \sum_{r=1}^{\infty} b_r\ \delta(t-2rT) \qquad (11)$$

Equation (11) gives the ideal reflection coefficient in the time domain.

The computer-aided design procedure compares the ideal response given by (11) with the actual circuit response given by (3). The values of the circuit elements are then adjusted to minimise the sum of the squares of the errors between the two functions.

Examples

Several circuits have been designed using this procedure; two examples are given below.

Example 1: The fifth order band-pass filter shown in Figure 3a was designed to approximate the ideal $S_{11}(t)$ shown in Figure 3b. The two responses were compared at the first 30 impulses. The resulting sum of the squares of the errors was 0.008 after 21 iterations and 35 seconds of computer time. The element values and the time and frequency responses are shown in Figure 3.

Example 2: The seventh and nineth order interdigital band-pass filters shown in Figure 4a were designed to approximate the ideal $S_{11}(t)$ shown in Figure 4b. The number of iterations was 17 and 27 and the computer times were 1.1. and 2.6 seconds respectively. The resulting sum of the squares of the errors was 0.0003 and 0.0002 respectively. The element values and the resulting frequency responses are given in Figure 4.

Conclusion

Time domain design offers an effective and efficient method for designing microwave networks. The derivation of the state and output equations using topological methods offers a general method of calculating the time response with no restrictions on the network topology.

References

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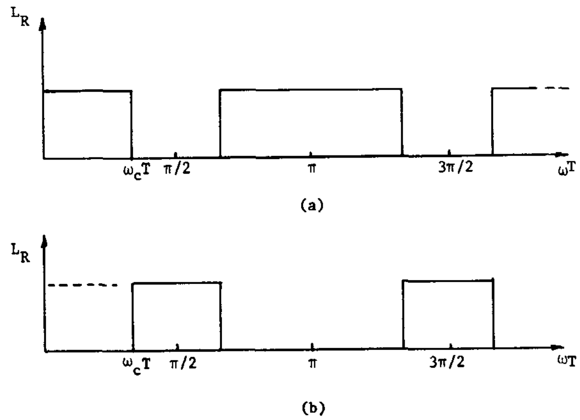


Figure 1: Ideal return-loss functions (a) low-pass (b) Band-pass

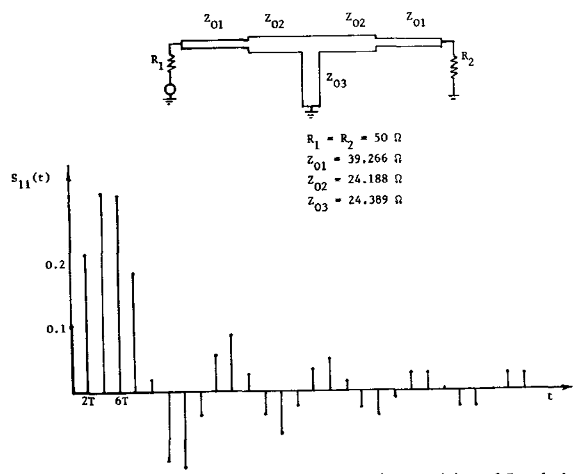


Figure 2(a): Circuit, element values and ideal reflection coefficient of Example 1.
A = 30 dB, $\omega_c T$ = 0.57.

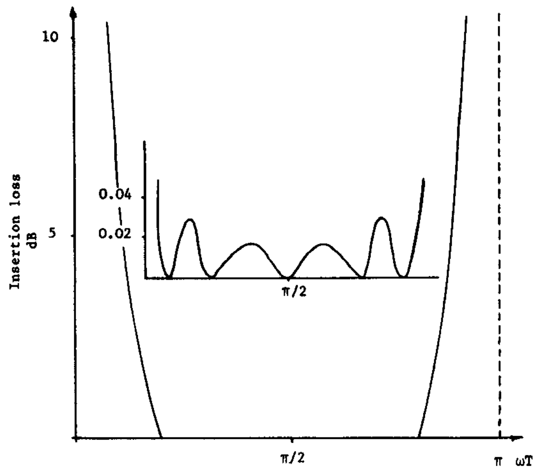


Figure 2(b): Frequency response of Example 1.

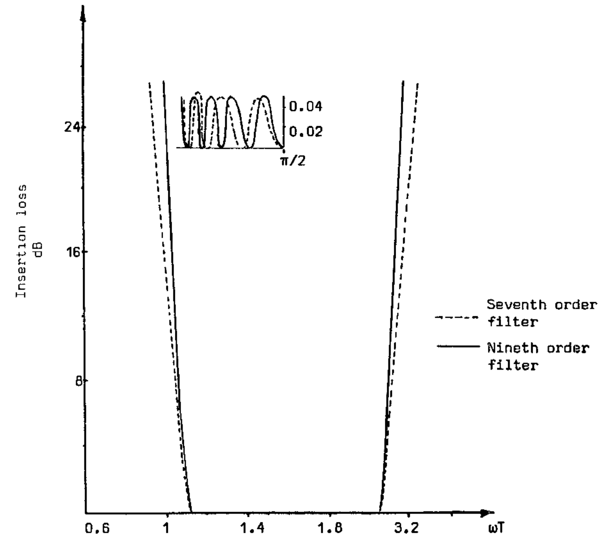
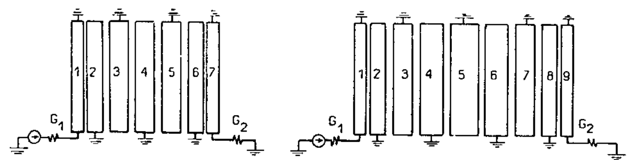


Figure 3(c) : Frequency response of band-pass filter (Example 2)



$G_1 = G_2 = 1$ mhσ
$Y_{11} = Y_{77} = 3.43605$ mhσ
$Y_{22} = Y_{33} = Y_{55} = Y_{66} = 4.5334$ mhσ
$Y_{44} = 4.948$ mhσ
$Y_{12} = Y_{21} = Y_{23} = \cdots = -1.1509$ mhσ

$G_1 = G_2 = 1$ mhσ
$Y_{11} = Y_{99} = 2.4313$ mhσ
$Y_{22} = Y_{33} = Y_{77} = Y_{88} = 4.568$ mhσ
$Y_{44} = Y_{66} = 5.0722$ mhσ $Y_{55} = 4.731$ mhσ
$Y_{12} = Y_{21} = Y_{23} = \cdots = -1.1548$ mhσ

Figure 3(a) : Networks and element values of Example 2.

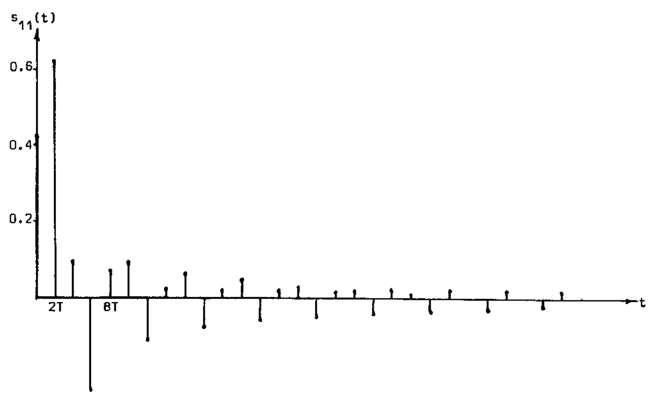


Figure 3(b) : Desired reflection coefficient response computed from ideal band-pass return loss, A = 25 dB, $\omega_c T = 1.0995$ (Example 2)